SuperRoot3

Andrew Robbins

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```
In [1]: # Dependencies
        tetration_dl = os.environ.get('TETRATION_DL', os.getcwd())
        sys.path.append(tetration_dl)
        from sage_tetration import *
```

1 The super cube root function

Defined as $SuperRoot3(z) = x$ if and only if $x^{x^x} = z$. It is usually written $\sqrt[3]{z}_s$.

```
In [2]: # Variables
        PS.<z> = PowerSeriesRing(QQ)
        x, y, k = var('x, y, k')

In [3]: superroot_3(0)

Out[3]: 0

In [4]: N(superroot_3(0.25))

Out[4]: 0.156692604084529

In [5]: N(superroot_3(0.5))

Out[5]: 0.367376852851984

In [6]: N(superroot_3(0.75))

Out[6]: 0.689537025983409

In [7]: superroot_3(1)

Out[7]: 1

In [8]: superroot_3(16)

Out[8]: 2

In [9]: superroot_3(3^3^3)

Out[9]: 3
```

1.1 Series of $SuperRoot3(z)$ at $z = 1$

```
In [12]: superroot_3._at1_series_(z, ring=QQ).O(10)

Out[12]: 1 + z - z^2 + 1/2*z^3 + 7/6*z^4 - 17/4*z^5 + 821/120*z^6 - 25/12*z^7 - 56269/2520*z^8 + 52079/

In [13]: superroot_3._at1_series_.egf[:10]

Out[13]: [1, 1, -2, 3, 28, -510, 4926, -10500, -900304, 26247816]
```

1.2 Series of $SuperRoot3(e^z)$ at $z = 0$

```
In [15]: superroot_3._exp_series_(z, ring=QQ).O(10)
```

```
Out[15]: 1 + z - 1/2*z^2 - 1/3*z^3 + 11/8*z^4 - 23/15*z^5 - 139/144*z^6 + 2677/420*z^7 - 10549/1152*z^8
```

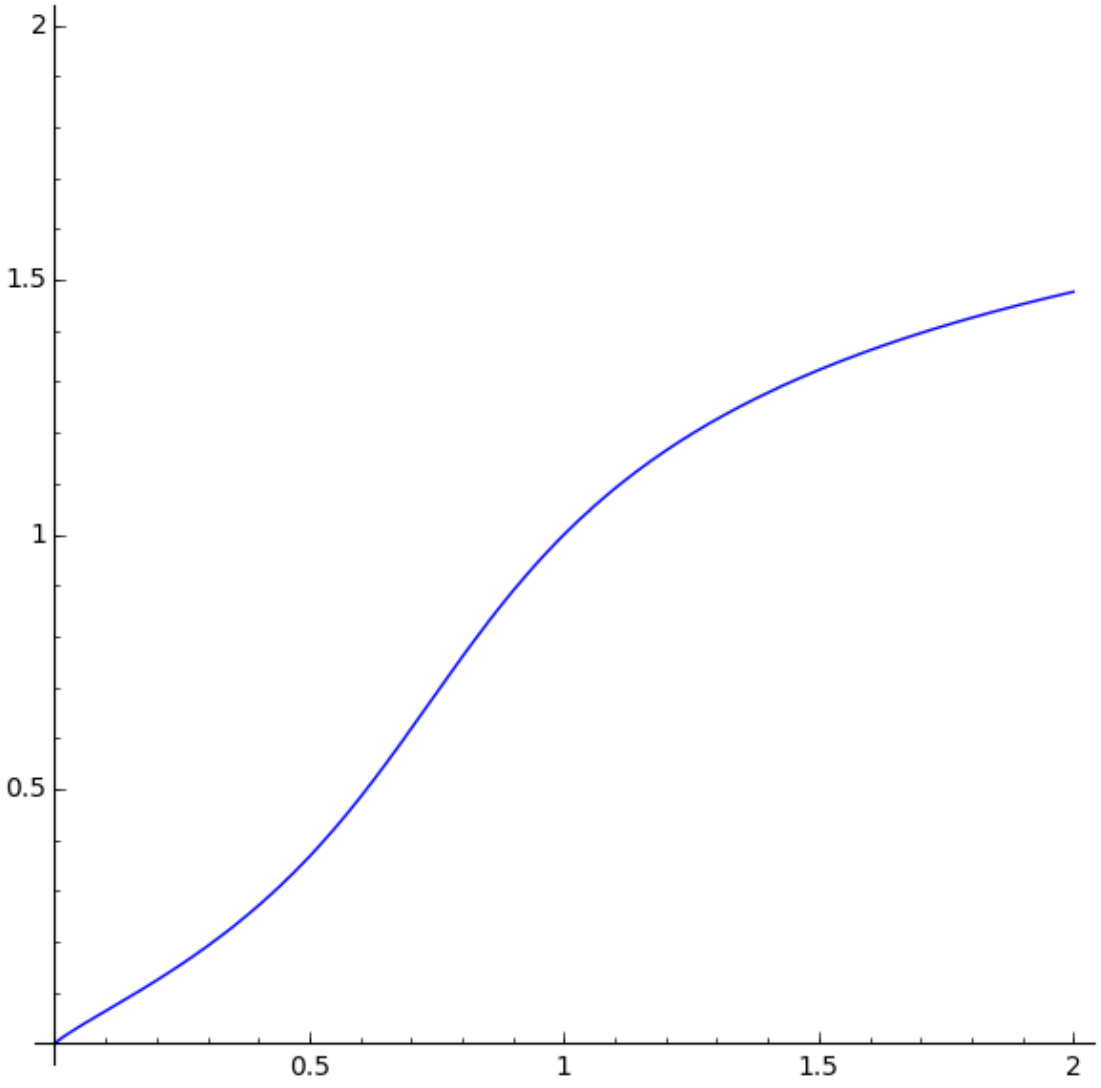
```
In [16]: superroot_3._exp_series_.egf[:10]
```

```
Out[16]: [1, 1, -1, -2, 33, -184, -695, 32124, -369215, -1298816]
```

1.3 Plots

```
In [17]: plot(lambda z: N(superroot_3(z)), (0, 2), ymin=0, ymax=2, aspect_ratio=1)
```

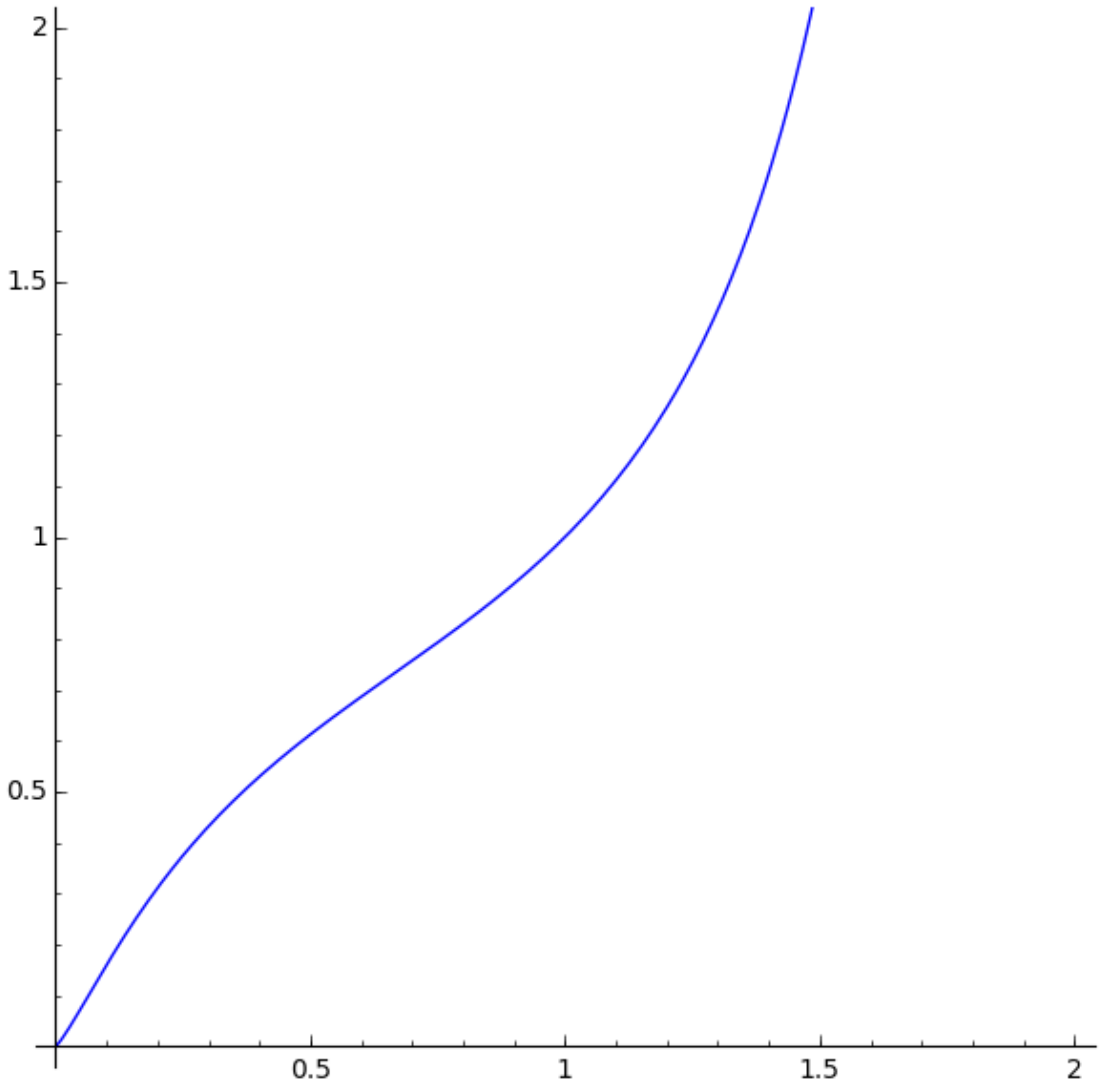
```
Out[17]:
```



This is the inverse function of $SuperPower3(z) = z^{z^z}$

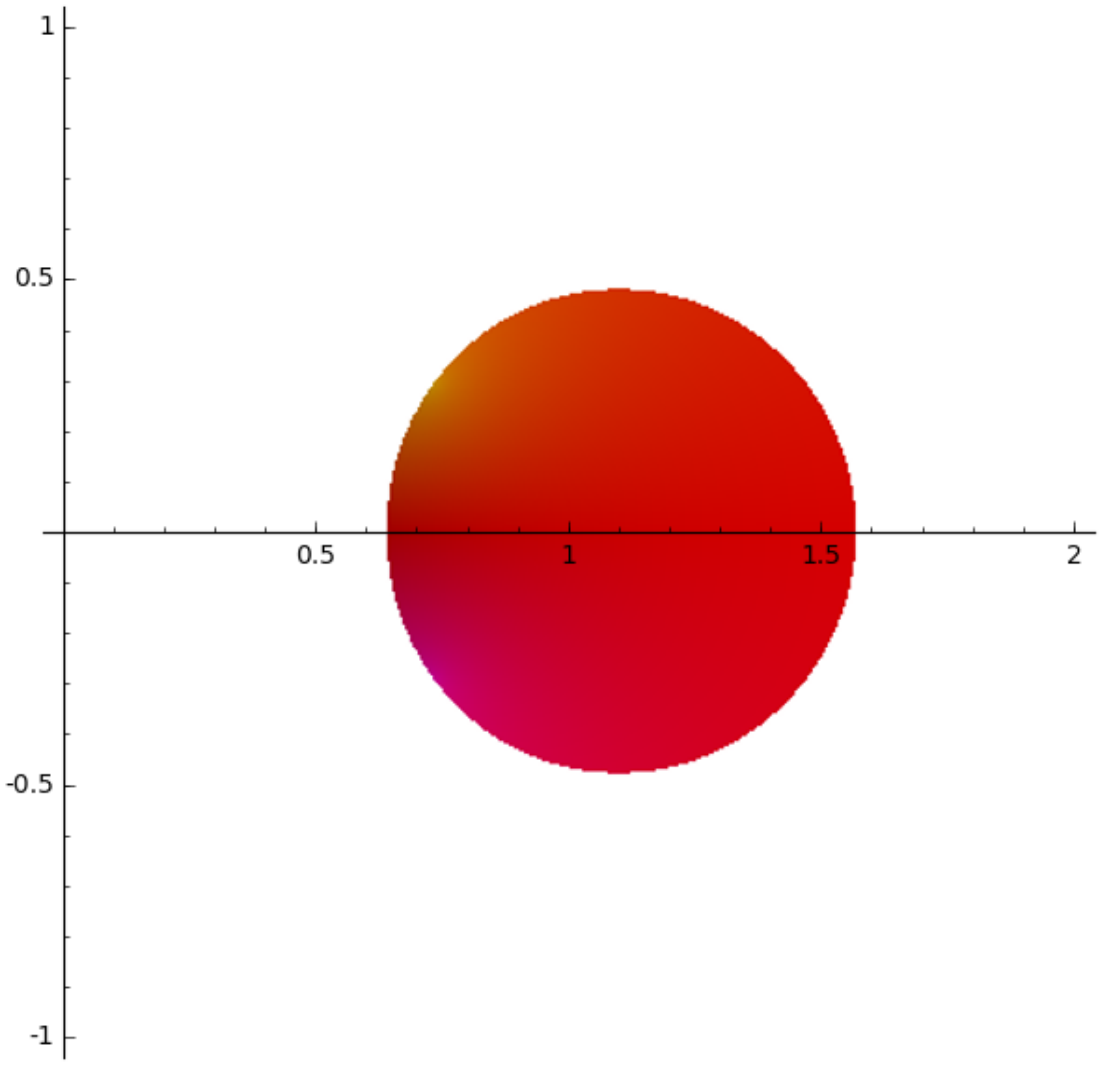
```
In [18]: plot(lambda z: z^z^z, (0, 2), ymin=0, ymax=2, aspect_ratio=1)
```

Out[18]:



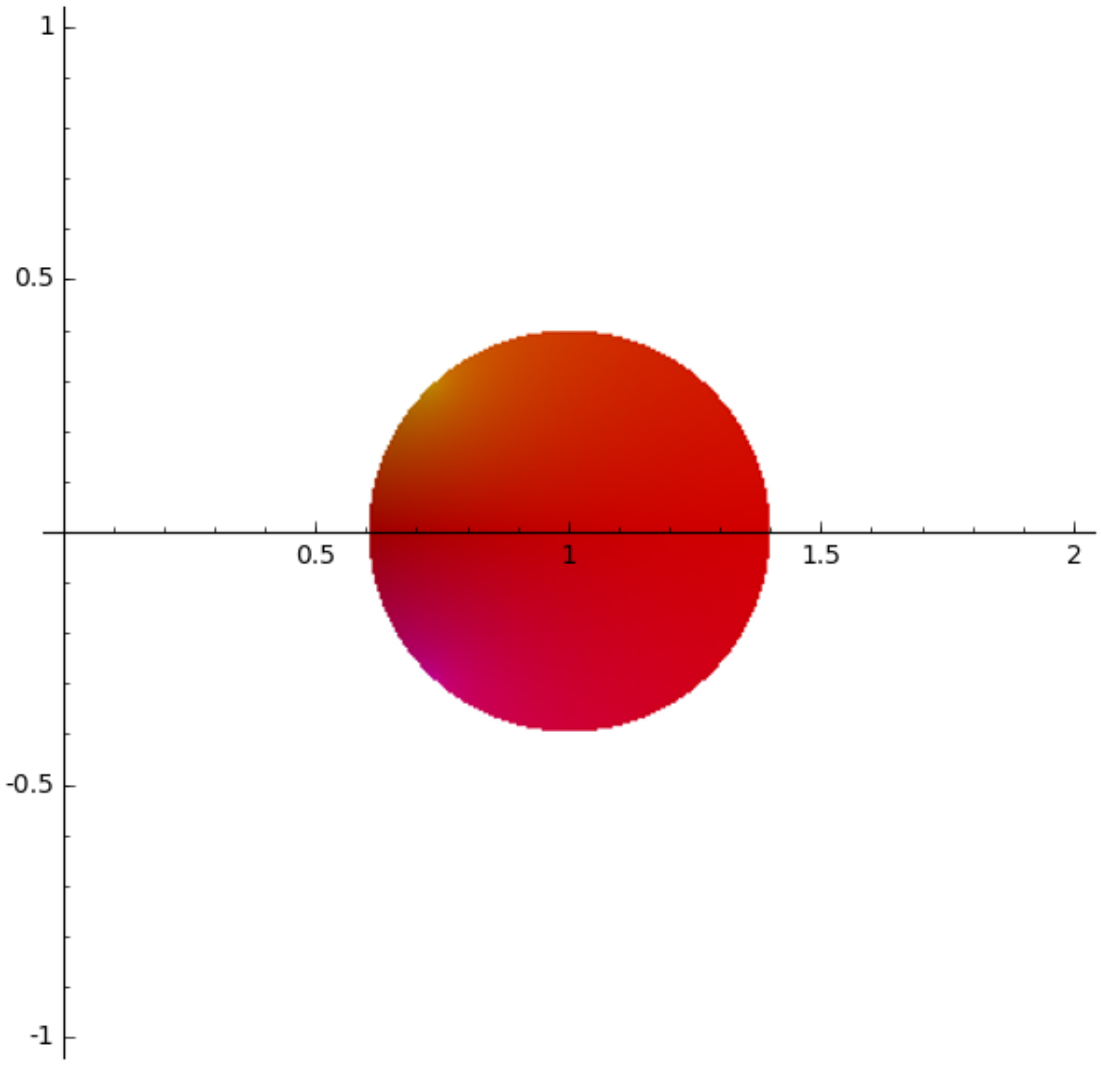
In [8]:
```
complex_plot(lambda z: superroot_3._exp_series_(N(log(z)), ring=CC) or infinity,
                    (0, 2), (-1, 1), aspect_ratio=1, plot_points=400)
```

Out[8]:



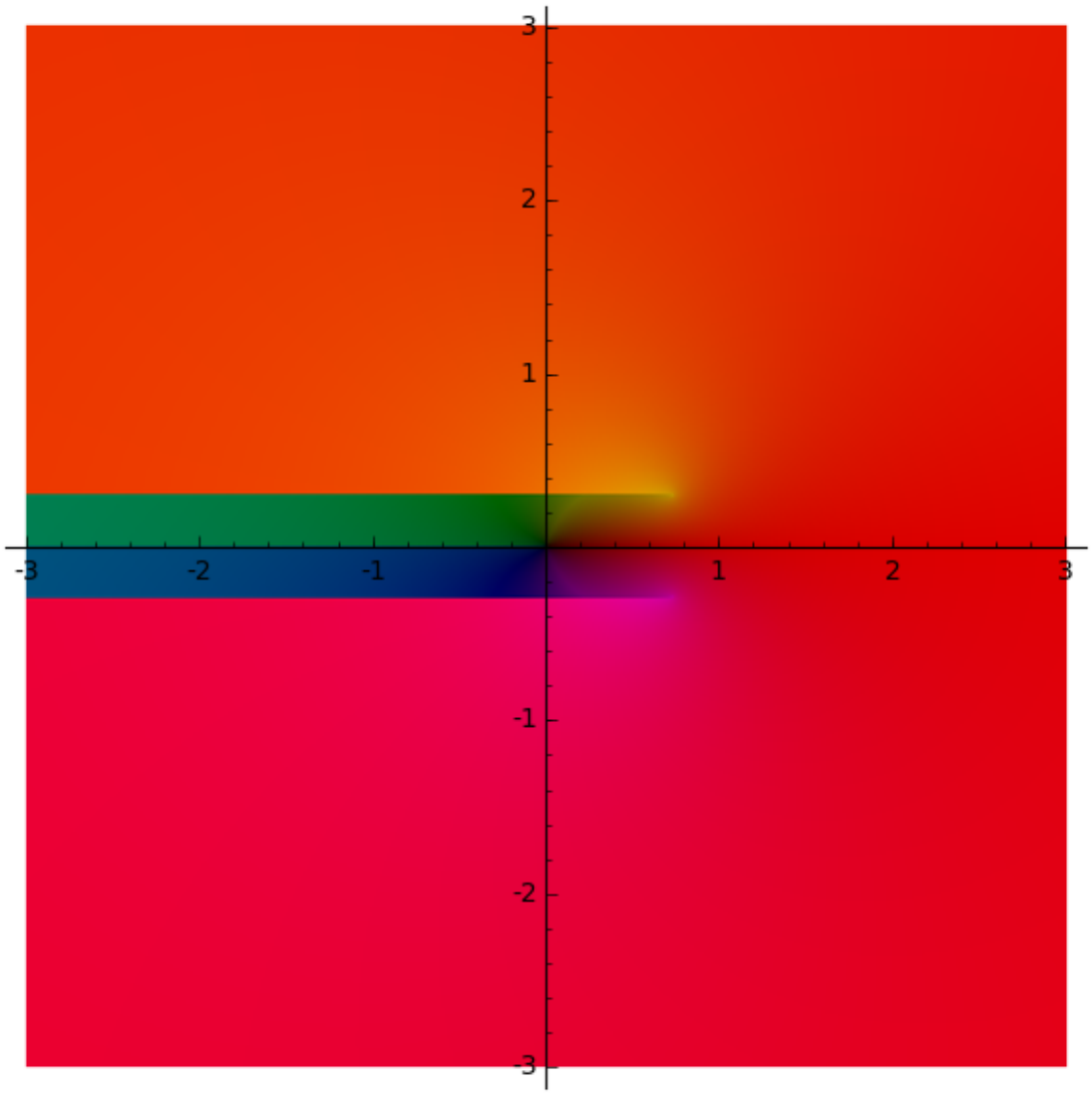

```
In [9]: complex_plot(lambda z: superroot_3._at1_series_(N(z), ring=CC) or infinity,
                     (0, 2), (-1, 1), aspect_ratio=1, plot_points=400)
```

Out[9]:



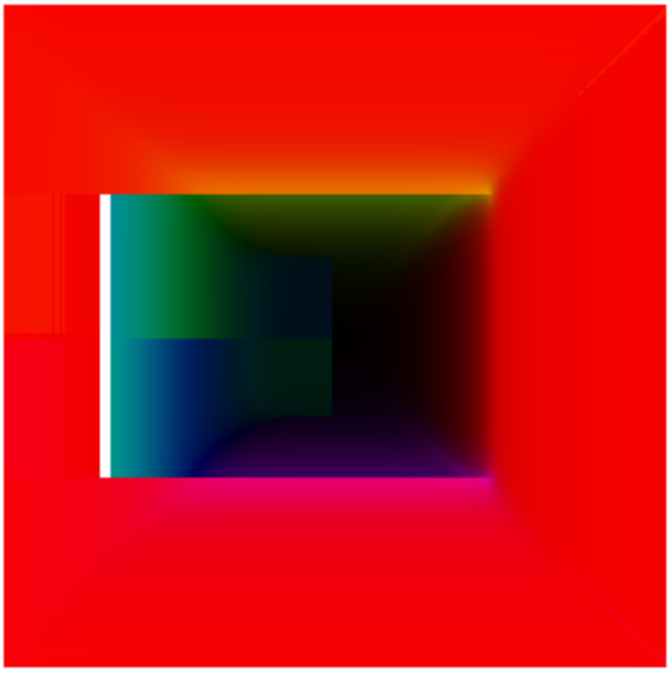

```
In [11]: complex_plot(lambda z: N(superroot_3(z)), (-3, 3), (-3, 3), aspect_ratio=1, plot_points=400)
```

Out[11]:



In [24]: float16_complex_plot(lambda z: N(superroot_3(z)))

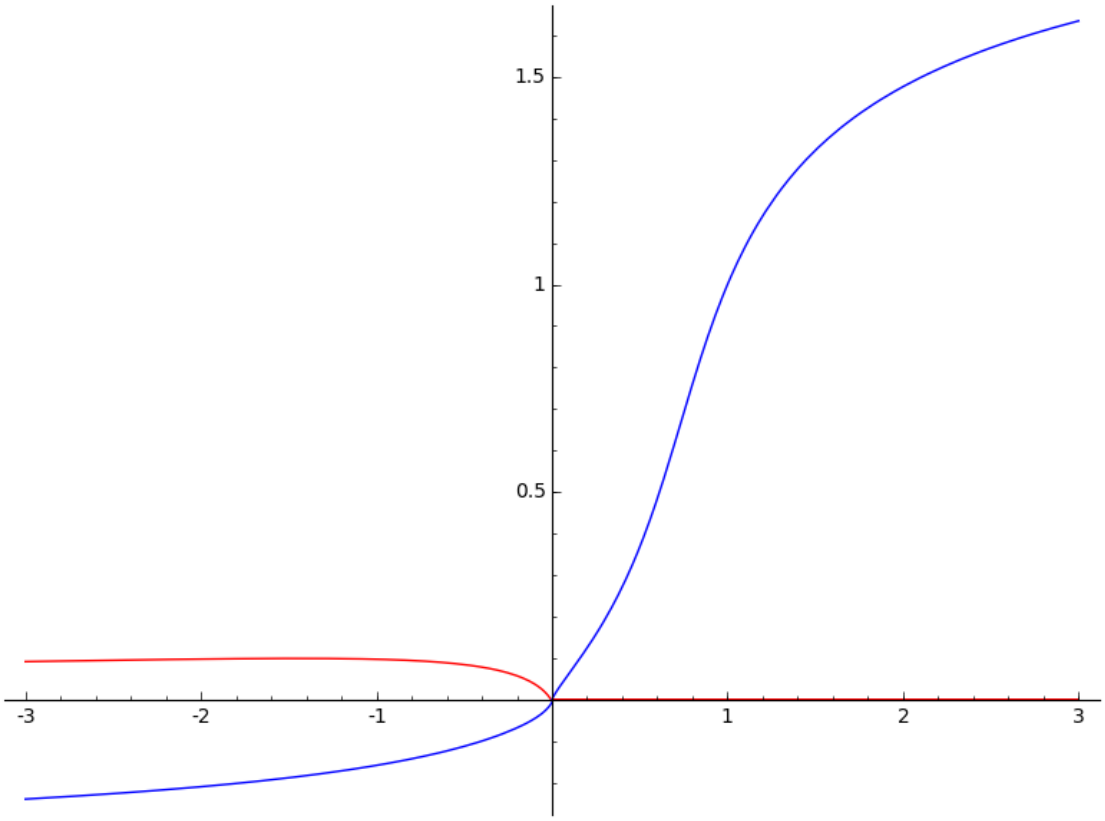
Out[24]:



```
In [25]: _.save(tetration_dl+"/superroot_3_float16_plot.png")
```

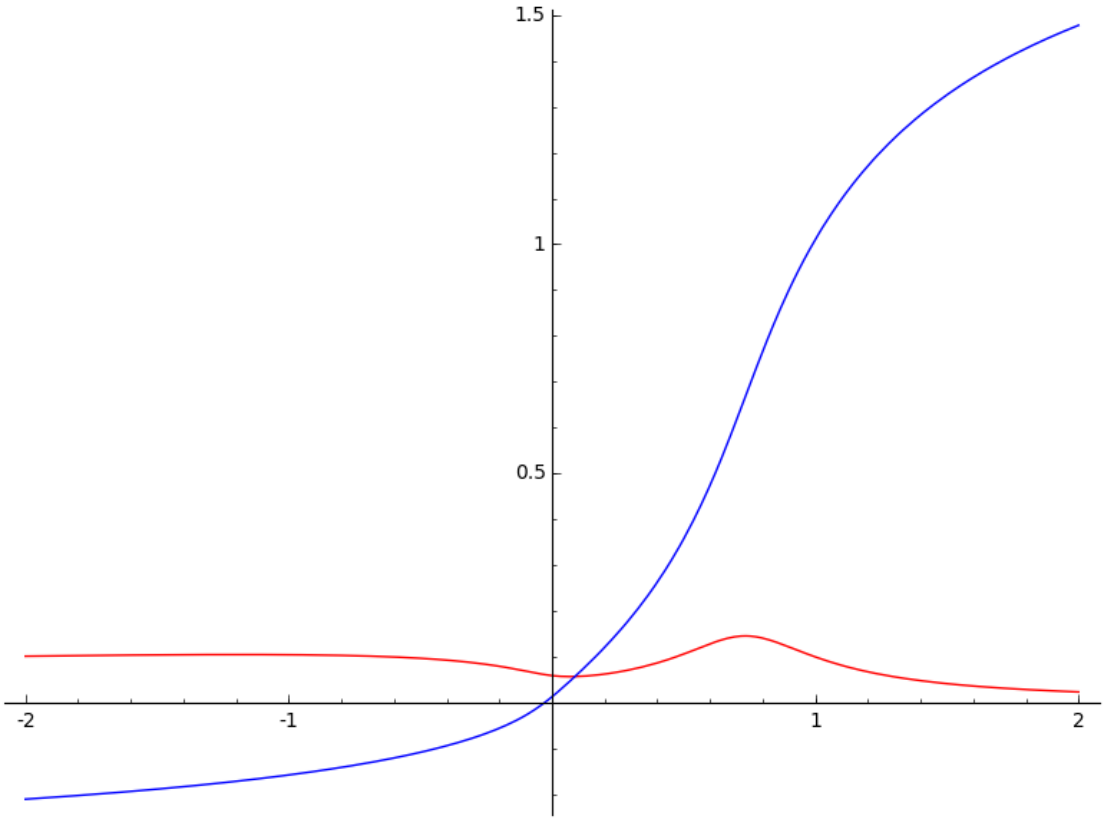
```
In [19]: real_complex_plot(lambda z: N(superroot_3(z)), (-3, 3))
```

```
Out[19]:
```



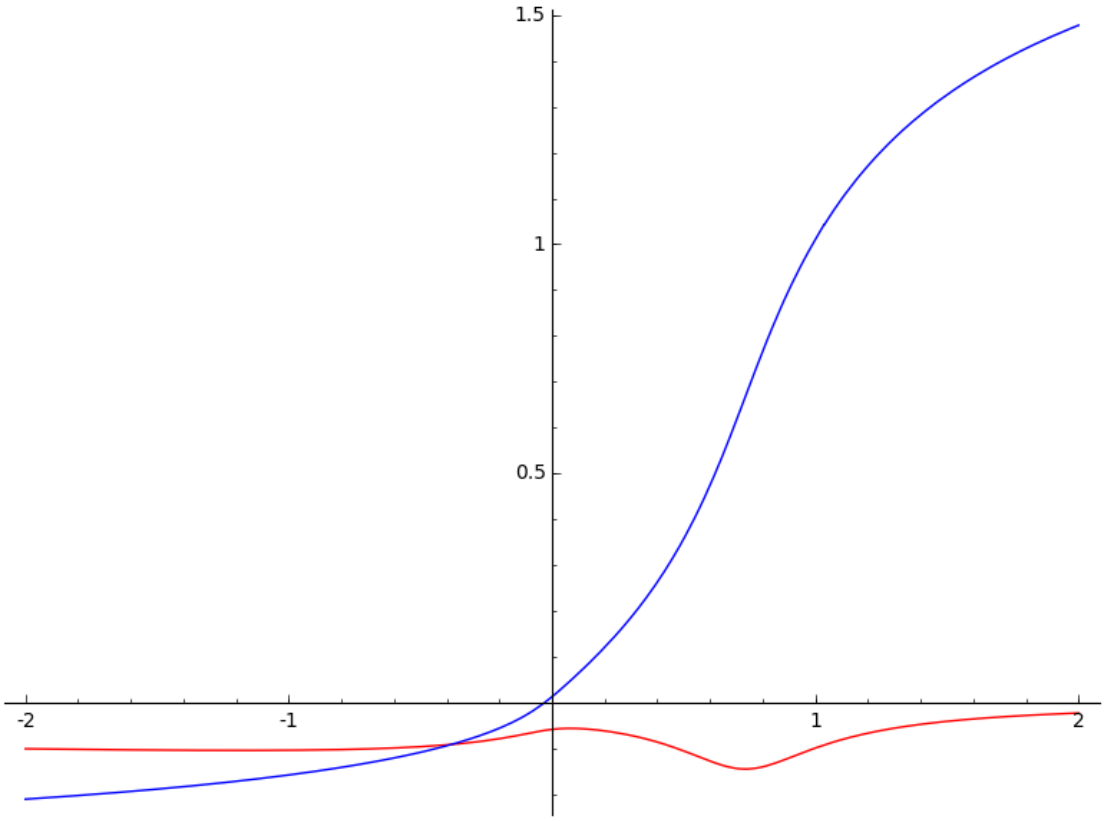

```
In [20]: real_complex_plot(lambda z: N(iterexproot_3(z + 0.1*I, 1)), (-2, 2))
```

Out[20]:




```
In [21]: real_complex_plot(lambda z: N(iterexproot_3(z - 0.1*I, 1)), (-2, 2))
```

Out[21]:




```
In [37]: isinstance(3, PowerSeries)
```

```
Out[37]: False
```

```
In [38]: isinstance(z, PowerSeries)
```

```
Out[38]: True
```

```
In [ ]: contours = [-1, -0.5, 0, 0.5, 1, 1.5, 2, 2.5, 3, 4]
```

```
In [23]: contours = list(filter(lambda x: (1/16 <= abs(x) and abs(x) <= 16) or abs(x) == 0,
                                map(float.fromhex, float16_iter8())))
         contours
```

```
Out[23]: [-16.0,
          -14.0,
          -12.0,
          -10.0,
          -8.0,
          -7.0,
          -6.0,
          -5.0,
          -4.0,
          -3.5,
          -3.0,
```

```
-2.5,
-2.0,
-1.75,
-1.5,
-1.25,
-1.0,
-0.875,
-0.75,
-0.625,
-0.5,
-0.4375,
-0.375,
-0.3125,
-0.25,
-0.21875,
-0.1875,
-0.15625,
-0.125,
-0.109375,
-0.09375,
-0.078125,
-0.0625,
-0.0,
0.0,
0.0625,
0.078125,
0.09375,
0.109375,
0.125,
0.15625,
0.1875,
0.21875,
0.25,
0.3125,
0.375,
0.4375,
0.5,
0.625,
0.75,
0.875,
1.0,
1.25,
1.5,
1.75,
2.0,
2.5,
3.0,
3.5,
4.0,
5.0,
6.0,
7.0,
8.0,
10.0,
```

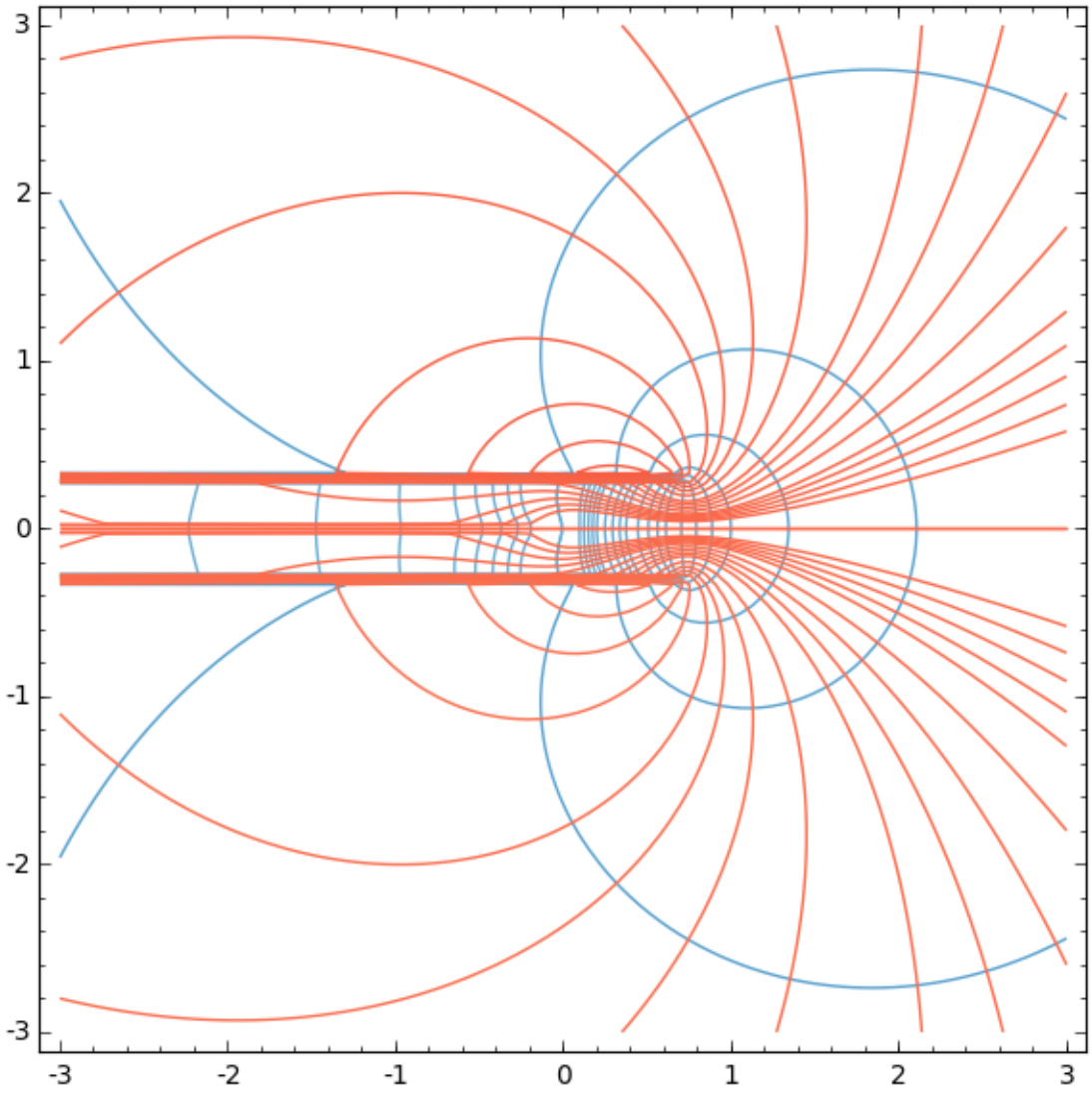
```
          12.0,
          14.0,
          16.0]
```

```
In [24]: preal = contour_plot(lambda x, y: N(real(iterexproot_3(0, N(x + y*I), 1))),
                              (-3, 3), (-3, 3), fill=False, cmap='Blues', contours=contours)
```

```
In [25]: pimag = contour_plot(lambda x, y: N(imag(iterexproot_3(0, N(x + y*I), 1))),
                              (-3, 3), (-3, 3), fill=False, cmap='Reds', contours=contours)
```

```
In [26]: preal + pimag
```

Out[26]:




```
In [40]: N(superroot_3(1.57175411534633 + 0.940677702091334*I))
```

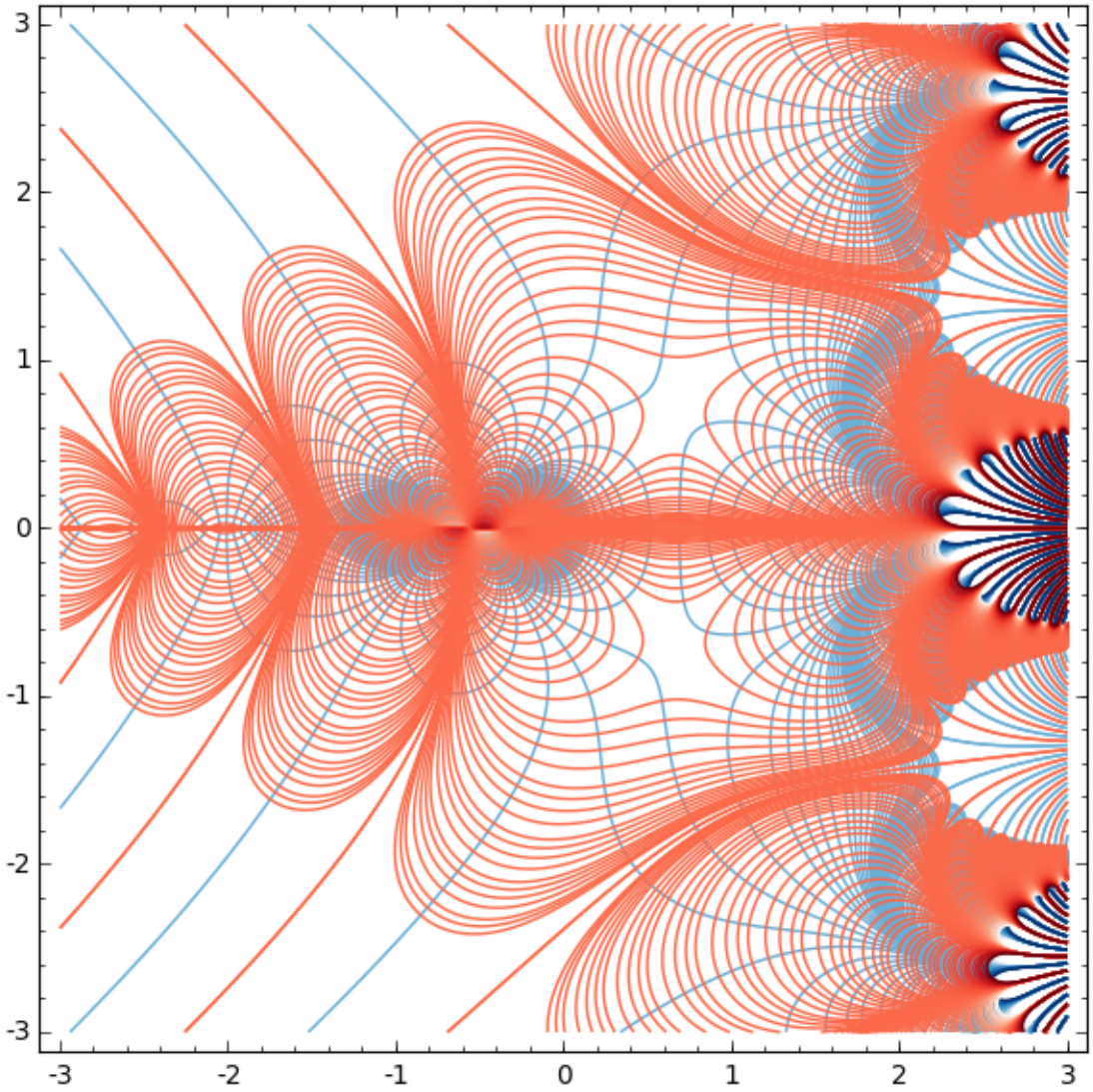
Out[40]: 1.50000000000000 + 0.250000000000000*I

In [39]:
```
iterexp(1.5 + 0.25*I, 3, 1)
```

Out[39]:
```
1.57175411534633 + 0.940677702091334*I
```

In [43]:
```
preal = contour_plot(
    lambda x, y: N(real(iterexp(x + y*I, 3, 1))),
    (-3, 3), (-3, 3), fill=False,
    plot_points=400, cmap='Blues',
    contours=contours)
pimag = contour_plot(
    lambda x, y: N(imag(iterexp(x + y*I, 3, 1))),
    (-3, 3), (-3, 3), fill=False,
    plot_points=400, cmap='Reds',
    contours=contours)
preal + pimag
```

Out[43]:



In [ ]:

In [ ]:

In [ ]: